Explicit Storage and Analysis of Billions of States using Commodity Computers

Yin Wang ∗ Jason Stanley ∗∗ Stéphane Lafortune ∗∗

∗ Hewlett-Packard Laboratories, Palo Alto, CA, USA
(e-mail: yin.wang@hp.com)
∗∗ Department of Electrical Engineering and Computer Science,
University of Michigan, Ann Arbor, MI, USA
(e-mail: {jasonsta,stephane}@umich.edu)

Abstract: The objective of this paper is to develop a framework and associated algorithms for explicit state space exploration of discrete event systems that can scale to very large state spaces. We consider classes of resource allocation systems (RAS), where a set of resources are shared by concurrent processes. In particular, we focus on Gadara RAS, whose Petri net representations have recently been used for liveness enforcement in multithreaded software. We present a framework where each reachable state of the RAS is represented by a single bit. We show how single-bit representations can lead to efficient implementations of supervisory control algorithms. In order to support single-bit state representations, we develop two indexing functions that map each state to a unique integer that serves as the corresponding index of the state in the large bit array. These functions exploit the invariants of the given RAS. Experimental results show that our techniques scale up to exploration and analysis of billions of states on commodity computers.

Keywords: Resource Allocation Systems, Petri nets, Supervisory Control, State Space Exploration

1. INTRODUCTION

A Resource Allocation System (RAS) consists of a set of resource types and a set of process types (Reveliotis, 2005). Each process type has multiple stages, organized according to the execution logic. Each stage requires a combination of resources to complete. A state of an RAS consists of one or multiple instances of different process types running at different stages. A taxonomy of various types of RAS is presented in Reveliotis (2005), based on the process execution logic and the resource allocation scheme. Petri nets are often used to represent RAS. There are various classes of Petri nets defined for different types of RAS; see (Ezpeleta and et al, 1995, 2002; Liao and et al, 2012a). The dynamics of an RAS can be captured by a finite state automaton (Nazeem and Reveliotis, 2011), which is equivalent to the reachability graph obtained from the Petri net representation. While the study of RAS originated in manufacturing systems, theoretical results on RAS have recently been applied to computer systems (Wang and et al, 2009).

The objective of this paper is to develop a framework and associated algorithms for efficient state space exploration of RAS that can scale to very large state spaces. The necessity of performing state space exploration of RAS occurs in several problem contexts, such as liveness enforcement. Liveness enforcement has been a central research theme in the RAS literature in the past two decades. Due to the state explosion problem, early solutions focused on structural properties using the Petri net representation in order to avoid state space exploration. These solutions typically sacrifice maximal permissiveness for computational efficiency. The survey paper Li et al. (2008) mentions that the only maximally permissive solution is based on the Theory of Regions (Ghaffari and et al, 2003), which requires exploring all reachable states, and then uses linear programming to synthesize a control place for each unsafe state. With the complete knowledge of the reachable state space, an alternative solution recently proposed in Nazeem and Reveliotis (2011) uses decision trees to classify safe and unsafe states and obtain a maximally permissive solution. For a special class of Petri nets called Gadara nets, structural analysis can be used to obtain a maximally permissive solution by bookkeeping unsafe states in the form of coverings (Liao and et al, 2012b), which can be proportional to the number of reachable states. Since determining whether a state is safe or not is an NP-complete problem even for one of the simplest classes of RAS (Reveliotis, 2005), an exponential complexity algorithm is unavoidable for provably achieving maximally permissive control (unless $P=\text{NP}$).

Model checking methods have been applied to systems with astronomical numbers of states (Burch and et al, 1992). These methods do not store each state explicitly, but instead rely on state reduction and compression techniques such as partial order reduction and symbolic state representation to achieve scalability. These techniques have been applied to generating Petri net state
In this paper, we present a framework and carefully crafted state manipulation algorithms that can explicitly store and analyze billions of states efficiently using a commodity computer. Our proposal is based on the idea of using a single bit to represent a state, which we show can lead to efficient implementations of state exploration and supervisory control algorithms. The key difficulty of this single-bit representation is the indexing function that maps each state to a unique integer serving as the index of the state. In the large bit array, Exploiting the invariants of certain classes of RAS, we develop two indexing functions with different space-time tradeoffs. In addition to these novel indexing functions, our implementation applies numerous optimization techniques. Most notably, we implement multithreading to achieve almost linear speedup in a multicore computer. Under this bit-representation framework, we further implement the basic supervisory control algorithm that calculates the supremal controllable nonblocking substate of states with respect to partial controllability. When applied to randomly generated RAS with a billion states, our program finishes state exploration and control synthesis in a few hours using less than 5GB of memory on an iCore 7 desktop.

The ability to analyze and control RAS in the billion states range is a novel contribution of this paper, as we are not aware of other methods for synthesizing maximally permissive solutions that can handle such large state spaces in a reasonable amount of time using commodity computers. Our tool, along with the random Gadara RAS generator that we use in this paper, are available open source at Gadara (2012).

This paper is organized as follows. We start with a description of the RAS model and a presentation of the basic search algorithm in Section 2. We then present our main results regarding the single-bit state representation and associated mapping functions in Section 3. Section 4 gives a brief description of our implementation, while Section 5 presents experimental results that demonstrate the scalability properties of our approach.

2. SYSTEM MODEL AND SEARCH ALGORITHMS

2.1 Resource Allocation Systems

Definition 1. (Reveliotis, 2005) A Resource Allocation System is defined as a 4-tuple \( \Phi = (R, C, P, D) \) where:

1. \( R = \{r_1, ..., r_m\} \) is the set of resource types.
2. \( C : R \to \mathbb{Z}^+ \) defines the capacity of each resource type, which we abbreviate as \( C(r_i) \equiv C_i \).
3. \( P = \{P_1, ..., P_o\} \) is the set of process types. Each type \( P_j \) is a composite \( (S_j, G_j) \), where \( S_j = \{S_{j1}, ..., S_{jl}\} \) is the set of processing stages, and \( G_j \) is the sequential logic that governs the execution of any process instance of type \( P_j \).
4. \( D : \bigcup_{j=1}^{n} S_j \to \prod_{i=1}^{m} \{0, ..., C_i\} \) is the resource allocation function associating every processing stage \( S_{jk} \) with an \( m \)-dimensional resource allocation vector \( D(\Xi_{jk}) \) required for its execution.

We aggregate all processing stages of all process types, and order them sequentially as \( \Xi_1, ..., \Xi_\xi \). A state of RAS \( \Phi \) is then a \( \xi \)-dimensional vector \( s \), where each component

spaces (Wolf, 2007). However, the effectiveness of state reduction and compression heavily depends on the redundancy and symmetry present in the system. When applied to “minimalist” RAS, symbolic model checking scaled up to only a few millions of states in our previous study (Wang and Wu, 2003). Coincidentally, other application domains report scalability limits in the million state range as well (Cimatti and Roveri, 2000; Jhala and Majumdar, 2009). The computation time is almost always proportional to the number of states.

It is well known that memory space rather than computation time is the bottleneck for explicit state enumeration (Wolf, 2007). The fundamental challenge for efficient storage is the data structure used to store and locate a state. The latter is needed in order to skip states already visited during the search process. One common technique is to use a conflict-free hash function to store each state in a compressed form (Wolf, 2007). Known hash functions are typically very inefficient for RAS since they are designed for generic Petri nets. In addition, these functions are designed for efficient compression, not uncompression. The latter is needed in order to skip states already visited.

Invariant

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In this approach does not scale up to a billion states because it needs to explicitly store all states satisfying invariant constraints. For example, using the random RAS generator that we have constructed, a system of a billion reachable states typically has more than 200 stages. The number of states satisfying invariant constraints is typically twice of the reachable states, i.e., two billion. These systems have unit resource capacity so each stage has at most one process.

Therefore we can use 200 bits to represent a state, or 25 bytes. The memory space for two billion states is 50 GB. For general disjunctive/conjunctive (D/C)-RAS with larger resource capacities, terabytes of memory are needed if we use a 32-bit integer to record the number of instances at each stage. External storage is not a viable option either due to its high latency. For example, a NAND-based Solid State Disk has a read latency of at least 25\(\mu\)s due to its physical characteristics. Assuming zero read time, locating a state in a billion entry array requires around 30 accesses, or 750\(\mu\)s. It would therefore take 208 hours to explore one billion states even if we query each state just once. Using mechanical disks would take years. For maximally permissive control solutions, experiments reported in the literature are typically limited to a few million states or less (Li et al., 2008; Nazeem and Reveliotis, 2011; Nazeem and et al, 2011; Liao and et al, 2011).

In this paper, we present a framework and carefully crafted state manipulation algorithms that can explicitly store and analyze billions of states efficiently using a commodity computer. Our proposal is based on the idea of using a single bit to represent a state, which we show can lead to efficient implementations of state exploration and supervisory control algorithms. The key difficulty of this single-bit representation is the indexing function that maps each state to a unique integer serving as the index of the state in the large bit array. Exploiting the invariants of certain classes of RAS, we develop two indexing functions with different space-time tradeoffs. In addition to these novel indexing functions, our implementation applies numerous optimization techniques. Most notably, we implement multithreading to achieve almost linear speedup in a multicore computer. Under this bit-representation framework, we further implement the basic supervisory control algorithm that calculates the supremal controllable nonblocking substate of states with respect to partial controllability. When applied to randomly generated RAS with a billion states, our program finishes state exploration and control synthesis in a few hours using less than 5GB of memory on an iCore 7 desktop.

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Each resource type defines an invariant, i.e., for each state of the automaton, the number of units occupied by all running process instances plus the vacant units must be equal to the capacity of that resource type. Therefore, every state must satisfy the following inequality:

$$\forall i = 1, \ldots, m, \sum_{k=1}^{\xi} s[k] \cdot D(\Xi_k)[i] \leq C_i$$  \hfill (1)

Using the terminology of supervisory control theory for regular languages represented by automata (see, e.g., Casandras and Lafortune (2008) for formal definitions), a state is accessible if it can be reached from the initial state $s_0 = [0, \ldots, 0]$, where there is no process instance running. A state is co-accessible if it can reach $s \in S_M$, where $S_M$ is the set of marked states; in RAS as above, the initial state $s_0$ is usually the only state that is marked. A deadlock state is a state without a successor. An RAS is live if every accessible state is co-accessible. An event of an RAS can be either controllable or uncontrollable.

Numerous classes of RAS have been proposed in the literature (Reveliotis, 2005). There are also many classes of Petri nets that capture different types of RAS. We present our ideas and results using a special class of RAS that arises from the modeling of multithreaded software, called Gadara RAS. Its precise definition is given in Nazeem and et al (2011), and its definition in Petri net form, called Gadara net, is given in Liao and et al (2012a). Here, we summarize the features of Gadara RAS/nets relevant to the design of our algorithms.

**Assumption 1:** The execution logic of each process type $P_j$ corresponds to a connected digraph where there is a one-to-one mapping between the node set and the stage set $S_j$. Every execution of an instance follows a path on the graph and there are no "AND-fork/join" stages.

**Assumption 2:** $\forall i = 1, \ldots, m$, $C_i = 1$, and $\forall k = 1, \ldots, \xi$, $D(\Xi_k) \in \{0, 1\}^m$.

Assumption 2 implies that each processing stage has at most one process instance. Therefore each state is a binary vector in $\{0, 1\}^\xi$. This is because in multithreaded software the resources are mutual-exclusion (mutex) locks that can be acquired by one thread at a time. The controlled variant of Gadara RAS (cf. Liao and et al (2012a)) allows additional resource types that need not be unit capacity. It can allocate more than one unit to a processing stage too, much like a semaphore in multithreaded programs. However, each processing stage requires at least one resource type of unit capacity, so the state is still a binary vector. Finally, the class of Disjunctive/Conjunctive (D/C)-RAS removes assumption 2 completely. We extend our solution to Controlled Gadara RAS and D/C-RAS in Section 3.4.

### 2.2 Generic State Search Algorithm

Algorithm 1 is the generic state search algorithm. It iteratively explores unflagged neighbors of the head of the queue, adds them to the queue, and then flag them. If the queue is first-in-first-out, e.g., a double queue, the algorithm is breadth-first search. If it is first-in-last-out, e.g., a stack, the algorithm is depth-first search. This generic search algorithm is instantiated into the following three operators on automata.

**Accessibility** acc($s_0$, $F$) calculates the set of states reachable from the initial state $s_0$ through a path that does not intersect $F$, the set of forbidden states. In this case, $s$.neighbor() is instantiated to calculate all successor states of $s$, i.e., forward search. If we start with an empty set of $F$, the algorithm returns all reachable states.

**Co-accessibility** co(acc($S_M$, $F$) calculates the set of states that can reach some state in the marked state set, $S_M$, through a path that does not intersect $F$. In this case, $s$.neighbor() is instantiated to calculate all predecessor states of $s$, i.e., backward search. 

**Uncontrollable Reach** uncon_reach($I$, $\emptyset$) calculates the set of states that can reach some state in $I$ through a sequence of uncontrollable transitions, including $I$. In this case, similar to the co-accessibility calculation, $s$.neighbor() is instantiated to calculate all predecessor states of $s$, but through uncontrollable transitions only.

### 2.3 Supremal Controllable Nonblocking Subset of States

We consider a simple form of supervisory control problem, where the control specification is avoiding a set of forbidden states rather than enforcing a regular language. Using the three operators described in the previous subsection, Algorithm 2 shows the iterative algorithm that calculates the supremal controllable nonblocking subset of states with respect to partial controllability (see, e.g., (Casandras and Lafortune, 2008)). When the input is $S_M = \{s_0\}, F = \emptyset$, Algorithm 2 returns the maximally-permissive liveness-enforcing solution for the RAS (Reveliotis, 2005).

### 3. SOFTWARE DESIGN

We present our main ideas using the class of Gadara RAS described in Section 2.1. The extension to D/C-RAS is...
After the calculation, marked bits in `java.util.BitSet` two subsections present two alternative mapping functions the computational complexity of the mapping. The next ratio the simplest mapping functions is to take the decimal value arrays of 10 billion entries takes less than half second using extremely fast. For example, calculating "AND" of two bit operations required for supervisory control in Algorithm 2
allocation array and the state space size as the all reachable states. We call the ratio between the size of the bit array is needed for this efficient storage. Since one-to-one mapping between the state set and the indices of the bit array is needed for this efficient storage. Since we do not know how many reachable states an RAS has to store and locate all visited states. A popular idea is to store each visited state in a compressed form (Wolf, 2007). Exploiting features of RAS, we push the idea to the extreme by effectively storing each state as one bit. A one-to-one mapping between the state set and the indices of the bit array is needed for this efficient storage. Since we do not know how many reachable states an RAS has before state exploration, the mapping function has to be conservative, and allocate a bit array large enough to store all reachable states. We call the ratio between the size of the allocated array and the state space size as the inflation ratio. There is a tradeoff between the inflation ratio and the computational complexity of the mapping. The next two subsections present two alternative mapping functions that optimize time and space, respectively.

Another advantage of the bit representation is that set operations required for supervisory control in Algorithm 2 can be implemented by logic bit operators that are extremely fast. For example, calculating “AND” of two bit arrays of 10 billion entries takes less than half second using a single thread on a 64-bit 2.6GHz iCore 7 CPU. Figure 1 shows the Java implementation of Algorithm 2 using bit operators, where `BitSet` is the standard JDK class `java.util.BitSet`. Marked bits in `S` represent reachable states, and marked bits in `F` represent forbidden states. After the calculation, marked bits in `S` are safe, while those in `F` are unsafe. We optimize the algorithm at lines 7-9 such that the uncontrollable reach is calculated for newly generated unsafe states only, at each iteration. This greatly reduces the computation time in our experiments where a significant portion of the reachable state space is unsafe. Finally, counting the number of “1”s of a bit array, or Hamming weight, is extremely efficient. The function `cardinality()` invoked at line 8 currently takes less than a quarter second to count “1”s in 10 billion bits. The latest SSE instruction set supports hardware Hamming weight calculation, which can further reduce the calculation time.

### 3.2 Cartesian Product Mapping

For Gadara RAS where each state is a binary vector, one of the simplest mapping functions is to take the decimal value of the binary vector as the index. However, this mapping is impractical for RAS because of its huge inflation ratio. For example, a Gadara RAS of 10 stages needs an array of $2^{10} = 1024$ bits. But if these stages all require one resource type, i.e., they are the support of the resource invariant, there are only 10 states. Based on this observation, we consider the state space defined by the Cartesian product:

$$||r_1|| \times ||r_2|| \times \ldots \times ||r_m||$$

where $||ri||$ denotes the support of the invariant defined by $ri$, i.e., set $\{ri\}$ union the set of stages that require resource type $ri$, $||ri|| = \{ri\} \cup \{\Xi_k \mid D(\Xi_k)[i] > 0\}$, or $D(\Xi_k)[i] = 1$ for Gadara RAS. A state $(\Xi_{s_1}, \ldots, \Xi_{s_m})$ in the Cartesian product space indicates that each resource type $ri$ is occupied by a process instance at stage $\Xi_{si}$. We include $ri$ in the support so (2) contains states that have unallocated resources. We slightly abuse the notation here and in the remainder of this section by using $\Xi$ for both stages and resources; note that both are `places` in the Petri net representation of an RAS.

The state space defined by (2) subsumes all states that satisfy the invariant constraint (1), and therefore it includes all reachable states. Our `Cartesian product mapping` scheme works as follows. We allocate one bit for each state defined by (2). Assuming $||ri||$ is an ordered set where $ri$ is the first element, the Cartesian product naturally defines a total order for the elements in its space, which we employ for the mapping. More specifically, a state $(\Xi_{s_1}, \ldots, \Xi_{s_m})$ in Cartesian space (2) is mapped to an index integer:

$I_1 + I_2 \cdot sizeOf(||r_1||) + I_3 \cdot sizeOf(||r_2||) + \ldots$  

where $I_k$ is the index of $\Xi_{si}$ in $||ri||$, $I_k = 0$ if $\Xi_{si} = r_i$, and $sizeOf(||ri||)$ is the cardinality of $||ri||$. We reverse the calculation to map an index integer back to a state; such a reverse calculation is straightforward to implement.

**Example 2.** Consider a Gadara RAS with two resource types, denoted by $a$ and $b$, and six stages, where $||a|| = \{a, \Xi_1, \Xi_2, \Xi_3, \Xi_4\}$, $||b|| = \{b, \Xi_4, \Xi_5, \Xi_6\}$. Applying our Cartesian product mapping to Example 2, we need an array of size $sizeOf(||a||) \cdot sizeOf(||b||) = 20$ bits. Each state in $||a|| \times ||b||$ is mapped to an index as follows: $(a, b) : 0, (\Xi_1, b) : 1, \ldots, (\Xi_4, b) : 4, (a, \Xi_4) : 5, (\Xi_1, \Xi_4) : 6, \ldots, (\Xi_4, \Xi_4) : 19$. Obviously all pairs $(\Xi_1, ?)$ and $(?, \Xi_4)$, 8 in total, correspond to only one state that satisfies the invariant constraint (1), which is the state containing only one instance at $\Xi_4$. In general, a state in (2) may not satisfy the invariant constraint (1) if there are stages requiring multiple resource types, i.e., $||r_1|| \cap ||r_2|| \neq \emptyset$. A variant of the Cartesian product mapping considers the following state space:
3.3 Decision Tree Mapping

The Cartesian product spaces defined by (2) and (3) are both larger than the solution space defined by the invariant constraint (1). The latter is very close to the solution space defined by the invariant constraint (1). The latter is very close to the resource invariant constraints are shown in the table of conditions is the initial state, therefore \( N(\emptyset) = 1 \). When \( R \neq \emptyset \), the set of states satisfying \( i \) and \( ii \) for \( R \) consists of states without any process instance on \( [R'] \), and states with process instances on \([R']\). In the latter case, there is exactly one process instance on \([R']\) because of Assumption 2. Since this instance takes all resources in \( R' \), there is no more process instance on any stage that requires resources in \( R' \). Hence we derive Equation (5).

Based on Equations (4-5), we construct a binary decision tree recursively for state mapping. We begin with \( R = 2^R \) (all of the power set) and the root node picks a set \( R_{root} \) randomly from \( R \). At the left child, \( R_{root} \) is removed from \( R \), which corresponds to the left term of (5). At the right child, any set that intersect with \( R_{root} \) is removed from \( R \), which corresponds to the right term of (5). The recursive process continues until \( R \) is empty. Every left child removes exactly one element from \( R \), while the right child may remove multiple elements. Therefore the depth of the tree is the length of the leftmost branch, which is always \( 2^{|R|} \). The element we pick to split each node, including \( R_{root} \), does not affect the tree depth. For a given Gadara RAS, we begin with only resource sets whose exact supports are not empty, instead of the entire power set \( 2^R \).

Figure 2 shows the decision tree for Example 2. The elements \( \text{set} \) and \( \text{support} \) inside each node correspond to \( R' \) and \([R']\) in equation (5). The detailed data structure is shown in Figure 3a. At the root node, we begin with \( R = \{(a), (a, b), (b)\} \). Picking \( \{a\} \) for decomposition, both \( \{a, b\} \) and \( \{b\} \) remain in the left node, while only \( \{b\} \) remains in the right node. The process continues until \( R \) is empty, and we add a dummy empty node at each leaf for the convenience of the recursive calculation.

In order to calculate the index for each state, we need to first find how many states each subtree represents, calculated by the \( \text{sum()} \) function in Figure 3a, which implements equation (5). The \( \text{sum()} \) value of the root node is the total number of solutions to the invariant constraint (1). For Example 2, the \( \text{sum()} \) values are shown on top of each node in Figure 2. We cache the value in memory for each node since it is heavily used by the mapping function, discussed next.

The index of a state is calculated by walking down the decision tree, shown as function \( \text{indexOF}() \) in Figure 3b. The function \( \text{node.getNodeSupportIndex(s)} \) at line 7 returns the index in \( \text{nodeSupport} \) if state \( s \) has a process instance on the corresponding stage, or -1 if \( \text{nodeSupport} \) is not occupied. For Example 2, all 13 stages that satisfy its resource invariant constraints are shown in the table of Figure 2 together with their mapped indices. Here a state is represented by an array of process stages that are occupied by process instances.

Our decision tree is not balanced since every left child removes exactly one element from \( R \), while the right child may remove multiple elements. The length of the left-most branch is the height of the tree, which in the
one process instance, and a state is not always a binary

each stage to be associated with a unit capacity resource

Disjunctive/Conjunctive (D/C)

negligible.

is only used temporarily during the search process, the

added resources types. Since the full state data structure

Algorithm 1 that must consider the availability of these

neighbor()

ture representing a state, and the

resource type can be fully recovered from this state repre-

each, the remaining units of each added non-unit capacity

is given as a set of stages occupied by one process instance

discussed so far in this section. The reason is that if a state

types and corresponding non-unity resource allocation

The extension from Gadara RAS to controlled Gadara

RAS (i.e., those with additional non-unit capacity resource
types and corresponding non-unity allocation weights) does not change any algorithm or data structure discussed so far in this section. The reason is that if a state is given as a set of stages occupied by one process instance each, the remaining units of each added non-unit capacity resource type can be fully recovered from this state representation. The only adjustments needed are the data structure representing a state, and the neighbor() function in Algorithm 1 that must consider the availability of these added resources types. Since the full state data structure is only used temporarily during the search process, the extra space needed to store the additional resource types is negligible. Disjunctive/Conjunctive (D/C)-RAS have the same process structure as Gadara RAS, but do not require each stage to be associated with a unit capacity resource type. Therefore, a stage can be occupied by more than one process instance, and a state is not always a binary vector. We can extend the Cartesian product mapping to D/C-RAS. In this case, the $i$-th element in the Cartesian product will still correspond to resource type $r_i$, but its set will include all possible ways to allocate $r_i$ to its support stages. Extension of the decision tree mapping technique to D/C-RAS is an open issue at this time.

4. IMPLEMENTATION DETAILS

As explained in Section 3, while we store each state as one bit, the full data structure must be used during the search phase in order to calculate predecessor and successor states. Algorithm 1 indicates that these unexplored states are stored in a queue. We discovered that depth-first search (using a FILO queue) results in a queue depth as much as half the size of the reachable state space. Storing half states in their full data structure form in memory would negate all the benefits of bit representation. Interestingly, breadth-first search (using a FIFO queue) reduces the depth to roughly one tenth of the state space size, but it is still too much to store in memory. Alternatively we can store only the neighbor that is going to be explored next in memory. In this case, the set of neighbors has to be recalculated each time we backtrack the search tree, which is a substantial amount of computation. We adopt an interesting idea reported in Wolf (2007), namely, we store only events in the queue instead of states. The only state in memory is the head of the queue. Moving up and down the search tree is realized by firing events backward and forward, respectively. Both can be calculated in linear time. Using depth-first search, we still store the search tree in a linear queue. A dummy event is inserted to indicate the boundary of each tree level. Currently, we store each event by a pointer to its data structure, which is 64 bits or 8 bytes in a 64-bit program. We plan to further optimize the computation space by sorting all events in an array and representing each event by its index using a minimum number of bits, e.g., 8 bits is sufficient for a RAS with no more than 256 events.

As we scale to the billion state range using the bit representation, computation time becomes more noticeable than computation space. For example, a billion bits occupy only 119MB of memory but their sequential exploration takes more than a day in our experiments. To mitigate this, we have implemented a parallel version of Algorithm 1, which is a non-trivial engineering task for depth-first search (Reif, 1985). Our idea is to let each thread maintain its own search tree, i.e., running Algorithm 1 separately. The starting state for exploration is stored in a shared work queue, which is initialized to the neighbors of the initial start state. A thread repeatedly grabs a state from the work queue and explores the state space on its own. Synchronization is done by the hardware-assisted command CompareAndSwap, which checks and updates a bit atomically. We also implemented a load balancing scheme, where each thread monitors the work queue and replenishes unexplored states from the top of its search tree whenever the queue is empty.

5. EXPERIMENTAL RESULTS

Our experiments are based on Gadara RAS randomly generated using our tool first reported in Wang and et al.
Garbage collection in Java.

The memory usage of the algorithm stored in the table is not fully accurate because of the memory on Case 7 and beyond. The memory usage for examples requiring more than 137 billion bits using array index in Java is 2^n because the maximum upper bound of 137 billion bits because the maximum.

In addition to the two mapping functions discussed in Section 3, we built a more “conventional” state exploration algorithm that does not utilize bit representations for the purpose of performance comparison; it is referred to as method “non-bit” hereafter. This algorithm stores each state using an array of pointers that point to stages with one process instance. We use this representation instead of a binary vector to model each state because the latter is very sparse in Gadara RAS. Storing only pointers to nonzero entries is more efficient. This pointer-based state representation also benefits the calculation of successor and predecessor states. We store all visited states in a hash table in order to locate them efficiently. Hash tables sacrifice computation space for speed.

Cases 1-10 in Table 1 are tested using a single thread on an iCore 7 3.07 GHz computer with 24GB of memory. The left side shows the complexity of the RAS and the right side shows the space and time needed by the three methods. The parameters we used to generate these random examples are available at Gadara (2012) so interested readers can repeat our experiments. Our examples have an unusually high percentage of unsafe states because they acquire resources in a random fashion. In contrast, a real-world RAS often follows certain rules for resource allocation, e.g., global ordering, so it is easier to understand and is less prone to deadlock. Our randomly generated examples represent worst-case inflation ratios for our algorithms. For Cartesian mapping, there are more stages that allocate multiple resources. For tree mapping, there are more states that satisfy the invariant constraints but are not reachable since there are too many deadlocks.

Overall, we observe that the decision tree mapping is most space-efficient among all three algorithms. Its inflation ratio is less than 3x in all cases. The inflation ratio of Cartesian mapping varies. Higher percentages of unsafe states often result in larger inflation ratios. Since our goal is to understand the performance tradeoffs of different methods, our implementation currently uses only one 64-bit integer array to store all bits. This creates an artificial upper bound of 137 billion bits because the maximum array index in Java is 2^31. Therefore, there is no result for examples requiring more than 137 billion bits using Cartesian mapping. The non-bit method requires memory space linear in the number of states, which runs out of memory on Case 7 and beyond. The memory usage reported in the table is not fully accurate because of garbage collection in Java.

The three columns of computation time represent the calculation of reachable states, supremal controllable non-blocking subset of states, and the reachable states in the controlled Gadara RAS. We use the control synthesis algorithm described in Liao and et al (2011) to calculate the controlled Gadara RAS for each case. This algorithm uses an MIP formulation to find siphons iteratively, to which we apply a timeout threshold of 30 minutes. Only the first six cases finish successfully. The number of additional resource types added in the controlled Gadara RAS is shown in the parentheses under the “resources” column. We are unaware of any published solution that can scale to billions of states for Gadara RAS. We note, however, that once 100,000 unsafe states are found, there is a “trivial” solution that avoids each (boundary) unsafe state by exactly one linear inequality constraint (Nazeem and et al, 2011). This could result in too many linear inequalities that require special handling. We do not evaluate our program against this type of controlled Gadara RAS. The computation time of all three algorithms is roughly proportional to the number of states. The non-bit algorithm is fast, especially with smaller examples, because it does not require state mapping. Decision tree mapping is the slowest because of its complex mapping algorithm. For Case 8, Cartesian mapping calculates reachable states but not unsafe states because it runs out of memory on the latter (needs three more bit arrays to store intermediate results).

Figure 4 shows the benefit of multithreading. There are six test cases. We run both tree mapping and Cartesian product mapping for all these cases using one to four threads on a four-core CPU, and each data point is the average of four runs. We can see that our multithreading is very effective. Finally, Cases 11-13 in Table 1 show three examples with more than a billion states using 16 threads on a dual 4-core (with hyperthreading) Xeon E5520 2.26Gz workstation with 96GB of memory. Exploring 2 billion states requires only a little over 5 hours on this workstation.

6. CONCLUSION

We have presented a framework and specialized algorithms that make it possible to explicitly generate and explore state spaces of RAS that have as many as billions of states, on a commodity computer. Under this framework, we have exploited structural properties of certain classes of RAS, known as Gadara nets and designed efficient mapping functions that enable the representation of each state by a single bit. Our algorithms not only perform...
### Table 1: Sample experimental results of state exploration using three different algorithms

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The generation of the reachable state space, but they also permit efficient analysis and control synthesis, such as the calculation of the supernal controllable sublanguage, a core operation in control synthesis problems. Our software tools are available open source at Gadara (2012).

**REFERENCES**


